Turbulent Flow in Two and Three Dimensions[1]

H. Tennekes
Royal Netherlands Meteorological Institute
de Bilt, Netherlands

Abstract

Three-dimensional turbulence occurs mainly in convective clouds and in the atmospheric boundary layer. Two-dimensional turbulence is a model for the statistical features of large-scale flows in the atmosphere. The differences between two- and three-dimensional turbulence are discussed, with a minimum of mathematics, in terms of elementary vorticity dynamics. The influence of the microstructure on the evolution of the large-scale features of the flow field is explored in some detail. A simple rationale is given for ignoring subgrid scale fluxes in numerical weather prediction.

1. Introduction

What do a numerical weather forecaster and a turbulence theoretician have in common? Are they not using the same equations of fluid dynamics and thermodynamics? Why is it that eddy fluxes play such a small part in the equations used for numerical weather prediction? Are eddies in turbulent flows unpredictable and is a statistical approach to turbulence phenomena unavoidable? Why can we follow the various stages in the life cycle of a middle-latitude cyclone by consulting successive weather maps, whereas we seem to be incapable of following the evolution of individual eddies in the atmospheric boundary layer?

Questions such as these deserve attention, and not just to satisfy our intellectual curiosity. This past winter we were forcefully reminded of the fact that modern technology does not make industrialized nations immune from climate fluctuations. If the atmosphere is indeed unpredictable for >10–14 days in the future, theories of climate and of climate changes will have to be based on a combination of statistics and dynamics, perhaps similar to the approach that has been used for many years in turbulence theory. The poleward eddy flux of angular momentum is a case in point: in a climate model that flux has to be parameterized in a way that is consistent with atmospheric dynamics, much like the downward eddy flux of momentum in the atmospheric surface layer has to be parameterized before a wind profile can be obtained. Turbulence theory now can explain many of the details of the effects of buoyancy on the "general circulation" in the surface layer; perhaps climate modelers can take some of their clues from turbulence modelers.

In the theory of the general circulation, mid-latitude storms may be thought of as so many eddies that participate in the maintenance of the poleward fluxes of momentum and heat. In a statistical approach these eddies lose their individuality; the problem of forecasting their evolution becomes irrelevant, because only their collective transport capabilities are of interest. From this perspective, climate dynamics becomes a part of turbulence theory of the general circulation. Turbulence is not likely to remain a specialty for micrometeorologists only!

2. How do we recognize flow patterns?

Both cumulus cloud and extratropical cyclone are familiar concepts for meteorologists (see Figs. 1 and 2). But why do we always sketch them like that? Why do we draw cumulus towers so as to suggest the presence of a very active microstructure? Is it simply in deference to cloud microphysicists, or is there some intrinsic dynamical significance to the turbulent microstructure of convective clouds? When we make a model of the life cycle of a cumulus congestus, would we dare—or even care—to predict the evolution of each wrinkle on its surface? If not, why not? Could it be that the microstructure of clouds, even if it is essential to their dynamics, is unpredictable?

The strange-looking "clouds" in Fig. 3 illustrate the issue by contrast. Clearly, an actively growing cumulus cloud without microstructure is not even recognizable as such. The role of the microstructure and the analytical methods by which it should be treated obviously deserve attention.

Take another look at the storm in Fig. 2. Why does it have no microstructure? Is that merely a matter of resolution? Is the resolving power of the synoptic observation network so poor that we are not able to see small-scale features, even if we want to? Obviously, there are severe resolution limitations in weather map analysis, but is this the crux of the issue? If we make a dynamical model of the life cycle of convective clouds, we include parameterizations of the statistical effects of their microstructure, through entrainment and exchange coefficients, subgrid scale eddy fluxes, energy dissipation rates, and the like. Why are those complications ignored in numerical prediction models for synoptic-scale flows? Why indeed is it that the eddy fluxes of the unresolved scales of motion do not occupy a prominent place in textbooks on dynamic meteorology?

It is necessary to be somewhat more specific. Numerical forecasting schemes usually include parameterized representations of the influence of the atmospheric boundary layer and of convective cloud ensembles on the evolution of weather patterns. The large-scale flow in the atmosphere interacts with the earth's surface through the

[1] This paper is adapted from a colloquium lecture presented at the University of Washington and at the Oregon State University in the fall of 1976.

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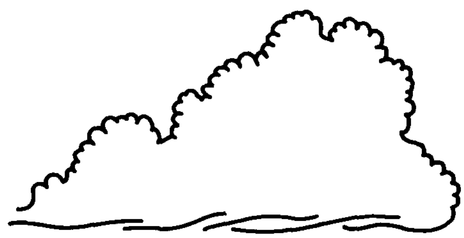


FIG. 1. A cumulus cloud is recognized most easily by its vigorous microstructure.

turbulence in the atmospheric boundary layer, and the vertical redistribution of heat and moisture is done, at least in part, by cumulus towers. But this is not the interaction we are concerned with here. The question is whether or not, apart from the effects of microscale turbulence, subsynoptic eddies above the friction layer do significantly affect the evolution of synoptic-scale systems.

In order to be fair to regional forecasters, we have to emphasize that we are discussing the *statistical* aspects of this problem. On occasion, subsynoptic eddies make dramatic contributions to weather changes, but in the context of this paper the question is whether or not their effects are strong enough *on the average* to require that they be accounted for in a systematic way. The microstructure of turbulence is known to be intermittent, with relatively intense small-scale eddies few and far between. Still, as we shall see shortly, the averaged effects of small eddies on the evolution of the boundary layer and of convective clouds cannot be ignored. The subsynoptic structure of atmospheric flows is also intermittent—rapidly intensifying smaller eddies interfere with forecasts only occasionally. Are subsynoptic eddies frequent and intense enough to require that their averaged effects be parameterized?

Again, it is useful to dramatize the point by contrast. The microstructure-filled "storm" in Fig. 4 would make a forecaster's task impossible. Even if most storms in fact look like the one sketched in Fig. 4 (and who has not wondered about the microstructure seen in many satellite pictures?), the task of predicting their evolution and migration apparently is not affected significantly by the eddy fluxes of the unresolved subsynoptic scales of motion. A typical subsynoptic eddy may change the time of frontal passage by half an hour, but there is no indication that subsynoptic eddies systematically advance or delay the time of frontal passage enough to warrant concern. It is worth emphasizing that the statistical problem is different from the deterministic one.

Our focus on the issue raised here can be improved by considering the diurnal cycle of the atmospheric boundary layer. The top of the boundary layer is the interface ("front") between the turbulence below and the free troposphere aloft. At any instant, a picture of this interface may look like the "fronts" sketched in Fig. 4. Boundary layer theory, however, ignores these wrinkles and defines the mixing height (depth of the mixed layer) as the *average* position of the interface. By the process known as entrainment or inversion erosion, the mixing height generally increases during the daylight hours, starting from a relatively low value at sunrise. Entrainment is caused by the unresolved scales of motion; models of the diurnal cycle of the mixed layer parameterize this effect in order to construct prognostic equations. In this particular example, the average position of the interface is determined entirely by the eddy fluxes of the unresolved turbulence; if these fluxes were absent, boundary layers could not change their height at all (except for the influence of large-scale horizontal convergence or divergence).

3. Vorticity in two-dimensional flows

How is it possible that the vigorous microstructure of turbulence does not have a counterpart in synoptic-scale flows? This, and all other questions we have raised, can be answered by studying the dynamics of vorticity in two- and three-dimensional flow fields. We begin with a brief review of the behavior of vorticity in synoptic-scale flow and take a frictionless, adiabatic, barotropic model without orography as the prototype of the kind of models used in numerical forecasting schemes. The flow in such a model is two dimensional: the velocity vector lies in the horizontal plane, and the vertical velocity component is zero. The vorticity vector, which is the curl of the velocity vector, then is perpendicular to the earth's surface at all times.

In frictionless, adiabatic, barotropic, nondivergent flow the absolute vorticity (the sum of the planetary and relative vorticities) is conserved along trajectories. The vorticity field is advected by the field of motion, and there is no mechanism to change the absolute vorticity of fluid parcels. No matter how complicated the flow patterns are at any given time, all they can do is redistribute the vorticity pattern.

Advection of vorticity, however, is not a trivial proposition. Figure 5 shows a sequence of pictures of dye advected around on the surface of a shallow dish filled with liquid (after Welander, 1955). The dye behaves in

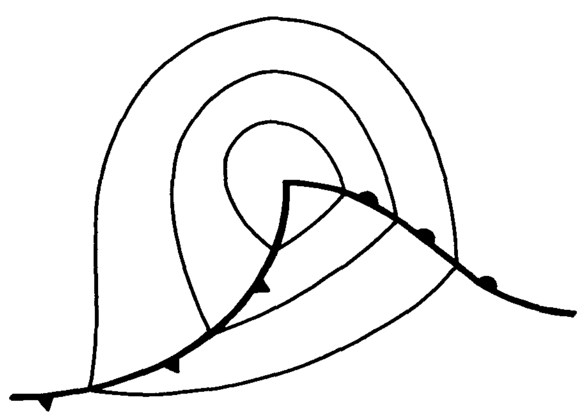


FIG. 2. A mid-latitude storm, shown here in a fairly early stage of its development, has smooth isobar patterns.

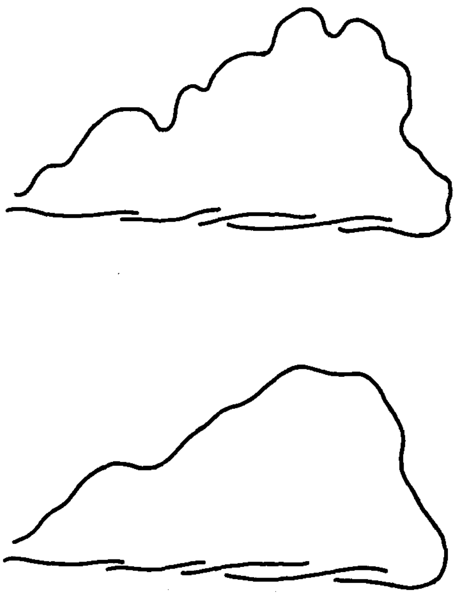


FIG. 3. As successive smoothing operations are performed on the outline of the cloud in Fig. 1, it becomes harder to recognize it as an actively entraining cumulus.

the same way as vorticity in two-dimensional flow fields: it is advected around without changing the dye concentration of small fluid parcels. The dye is conserved along trajectories. As time proceeds, the dye pattern (which thus may be viewed as marking all fluid parcels that have slightly higher or lower vorticity than the surrounding fluid) becomes ever more convoluted.

If the fluid has a finite viscosity, it is not hard to imagine how this process will eventually end: after sufficient time has elapsed, the filaments will become so thin that molecular diffusion (of dye or of vorticity) will begin to blur the contrast. This observation is the foundation of the theory of what is called two-dimensional turbulence. In turbulent flows, fluid parcels tend to drift away from each other by the effects of relative motion. As this happens, the length of filaments with slightly different vorticity (or of filaments marked with dye) tends to increase, and their width tends to decrease. This increases the vorticity contrast between neighboring parcels. In other words, "random" advection tends to drive vorticity variations toward smaller scales of motion, until the scales become so small that diffusion can take over. This is the core of the concept of the "enstrophy cascade" in two-dimensional turbulence (Leith, 1968).

Enstrophy is one-half of the mean-square vorticity in the flow field; it tends to move toward smaller scales of motion by the very nature of differential advection, and it is finally destroyed by molecular (or microscale turbulence) exchange processes at the smallest scales of motion.

The theory of two-dimensional turbulence is the specialty that deals with the statistical dynamics of adiabatic, barotropic, nondivergent models of atmospheric flow. In this theory, surface friction is often ignored, but internal friction (no matter how small) is not. In the absence of friction the enstrophy cascade would require an infinitely long time to reach infinitesimally small scales of motion. A truncated spectral model of two-dimensional turbulence without friction "reflects" enstrophy at the spectral cutoff, thus changing the dynamics of the flow field. Three-component models, for example, become periodic if friction is ignored (Lorenz, 1960; Dutton, 1976). In the absence of friction the flow would be thermodynamically reversible; the dissipation of enstrophy at the small-scale end of the spectrum makes the system irreversible. This makes sense not only from the point of view of the sequence in Fig. 5, but also from the perspective of climate dynamics. The "climate" of frictionless flow would not be a good model for the actual climate!

On the basis of what we have learned so far, it is not hard to estimate how the statistical parameters of the microstructure of two-dimensional turbulence depend on the scale of motion. We assume that the flow is driven by some substitute of baroclinic forcing at a scale L. Since we are concerned with subsynoptic eddies, we will not deal with the motion at scales larger than L (planetary waves and the like). The rate at which enstrophy moves toward eventual destruction at the small-scale end is given by the parameter χ. This parameter is called the "enstrophy cascade rate"; it is defined as the flux of enstrophy from one scale of motion to the next (Leith, 1968). The enstrophy cascade rate χ can be estimated as the ratio between the enstrophy at the principal scale of forcing and the characteristic time of eddies at that scale. If the typical velocity of eddies at scale L is designated as $v(L)$, the enstrophy at that scale will be proportional to $v^2(L)/L^2$, and the enstrophy cascade rate χ will be proportional to $v^3(L)/L^3$ (the time scale is proportional to $L/v(L)$ if the large-scale dynamics is controlled by inertia).

At each scale l such that $l < L$, but large enough to avoid appreciable viscous effects, the characteristic velocity $v(l)$ then presumably is a function of χ and l. By assumption, the effects of the parameters that define the nature of the forcing are absorbed into the cascade rate χ. By dimensional homogeneity, the only possible estimate for $v(l)$ becomes

$$v(l) \sim \chi^{1/3} l. \tag{1}$$

The vorticity fluctuations $\zeta(l)$ associated with "eddies" of scale l may be estimated as the typical velocity gradients associated with the relative motion at that scale. This yields

$$\zeta(l) \sim v(l)/l \sim \chi^{1/3}. \tag{2}$$

This estimate makes sense. We have seen that the vorticity of fluid parcels is conservative as long as friction plays no role; that implies that vorticity fluctuations should move toward smaller scales without changing their intensity. In other words, $\zeta(l)$ should be independent of scale for scales l smaller than L. This is exactly what (2) states. In the earth's atmosphere, $\chi^{1/3}$ is $\sim 1\ d^{-1}$.

The vorticity gradients at scale l can be estimated in the same way. We obtain

$$(\partial\zeta/\partial x)_l \sim \chi^{1/3} l^{-1}. \qquad (3)$$

This agrees with what we have seen in Fig. 5: although the vorticity (or dye concentration) of fluid parcels does not change as the cascade proceeds, the characteristic gradients continue to increase until they are so large that diffusion can take over.

The accelerations associated with eddies at "sub-synoptic" scales are of interest because they give an indication of the contributions of each scale to the accelerations with which the equations of motion are concerned. Taking a typical advective acceleration term, say $u\ \partial u/\partial x$, as our guide, we find

$$a(l) \sim v^2(l)/l \sim \chi^{2/3} l. \qquad (4)$$

We will return to the significance of these estimates after we have developed a corresponding set for three-dimensional turbulence.

The estimates (1)–(4) are consistent with the k^{-3} energy spectrum in the enstrophy-cascading range of two-dimensional turbulence (Kraichnan, 1967; Leith, 1968; Batchelor, 1969). The kinetic energy of eddies at scale l is proportional to $\chi^{2/3} l^2$; if we change from length scale to wave number with $kl = 1$, we obtain $\frac{1}{2} v^2(k) \sim \chi^{2/3} k^{-2}$. The "spectrum" (power spectral density) is the energy per unit wave number; this is clearly proportional to k^{-3}.

Spectra of this type are observed also in the earth's atmosphere, but there they arise not so much from the fact that atmospheric flow is two-dimensional (it is not!) as from the geostrophic constraints on flows at horizontal scales >100 km or thereabouts. Two-dimensional turbulence is a model for atmospheric motion in the sense that the vorticity in two-dimensional flow is subject to the same constraints as the pseudopotential vorticity in quasi-geostrophic atmospheric flow (Fjørtoft, 1953; Lorenz, 1960; Charney, 1971; Dutton, 1974).

4. Vorticity in three-dimensional flow

A fundamental feature of three-dimensional turbulence (such as found in convective clouds and in the atmospheric boundary layer) is that the vorticity of fluid parcels is not a conservative quantity. Suitable patterns of convergence and divergence create fields of differential vertical motion, and the vertical stretching of fluid parcels can amplify vorticity, much as in dust devils, tornadoes, and hurricanes. In turbulent flow that is neither confined to a plane nor subject to geostrophic constraints, this process takes place with a vengeance; vorticity fluctuations are amplified continuously at all scales of motion and in all three space dimensions. The total amount of enstrophy is extremely large if the viscosity of the fluid is sufficiently small.

Hurricanes, tornadoes, and dust devils concentrate kinetic energy into scales of motion that are smaller (often substantially smaller) than the scales of motion from which that energy is drawn. In the same way, turbulent flow tends to move kinetic energy toward smaller scales of motion, as a necessary consequence of the vorticity-amplification ("vortex stretching") process (Tennekes and Lumley, 1972). Since energy is a conservative quantity as long as the scales are too small for viscous dissipation to be effective, it makes sense to conceive of an "energy cascade" in which energy moves toward ever smaller scales toward eventual viscous dissipation. Following Kolmogorov, we denote the rate at which kinetic energy moves through the cascade by ϵ and assume that the typical velocity of eddies of any scale l that is smaller than the principal scale of forcing (but larger than the dissipative scales of motion) is determined by ϵ and l. Dimensional homogeneity then requires that

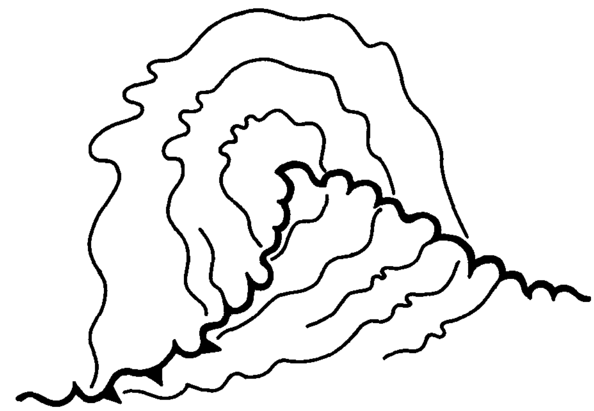


FIG. 4. The storm of Fig. 2 might look like this if it had microstructure dynamics similar to that of three-dimensional turbulence.

$$w(l) \sim \epsilon^{1/3} l^{1/3}. \qquad (5)$$

Here, we have used $w(l)$ to denote velocity differences over distances of order l; this helps to remind us that (5) is an estimate for three-dimensional turbulence, whereas (1) is an estimate of the eddy velocities in two-dimensional turbulence.

The velocity gradients and vorticity fluctuations of eddies at scale l now are estimated as $w(l)/l$, just as in the two-dimensional case. This yields:

$$\zeta(l) \sim w(l)/l \sim \epsilon^{1/3} l^{-2/3}. \qquad (6)$$

The amplitudes of the vorticity fluctuations grow larger as the scale of motion decreases, thus confirming that a conservative energy cascade must be associated with enstrophy amplification.

The accelerations associated with eddy motions of scale l are estimated as

$$a(l) \sim (u\ \partial u/\partial x)_l \sim w^2(l)/l \sim \epsilon^{2/3} l^{-1/3}. \qquad (7)$$

The amplitudes of these acceleration fluctuations increase as the scale of motion decreases, and the largest accelerations are encountered near the smallest scales of motion.

FIG. 5. Successive stages in the evolution of a patch of dye being advected around on the surface of a pan filled with liquid (after Welander, 1955). The original pictures were redrawn with a fairly wide Leroy pen in order to simulate the effects of a small, but finite, diffusivity.

The estimates (5)–(7) agree with the $k^{-5/3}$ spectrum in the inertial subrange of three-dimensional turbulence. According to (5), the kinetic energy (per unit mass) at scale l is estimated as $\frac{1}{2}w^2(l) \sim \epsilon^{2/3}l^{2/3}$. Switching from scale to wave number with the aid of $lk = 1$, we obtain $\frac{1}{2}w^2(k) \sim \epsilon^{2/3}k^{2/3}$. Since the spectral density is proportional to kinetic energy per unit wave number, the corresponding spectrum obeys a power law with an exponent equal to $-5/3$.

5. Microstructure dynamics

What is the significance of these estimates? We begin by considering the effects of the microstructure on the large-scale flow. If the flow field is decomposed into resolved and unresolvable scales of motion, the equations of motion for the resolved scales contain terms that represent the averaged effects of the unresolvable scales on the resolved ones (the averaging, of course, is associated with the resolution limitations involved). In the equations of motion, these terms take the form of a divergence of the (unresolved) eddy flux of momentum. The point here is that the divergence of this flux is the contribution that unresolvable eddies make to the accelerations in the resolved field of motion.

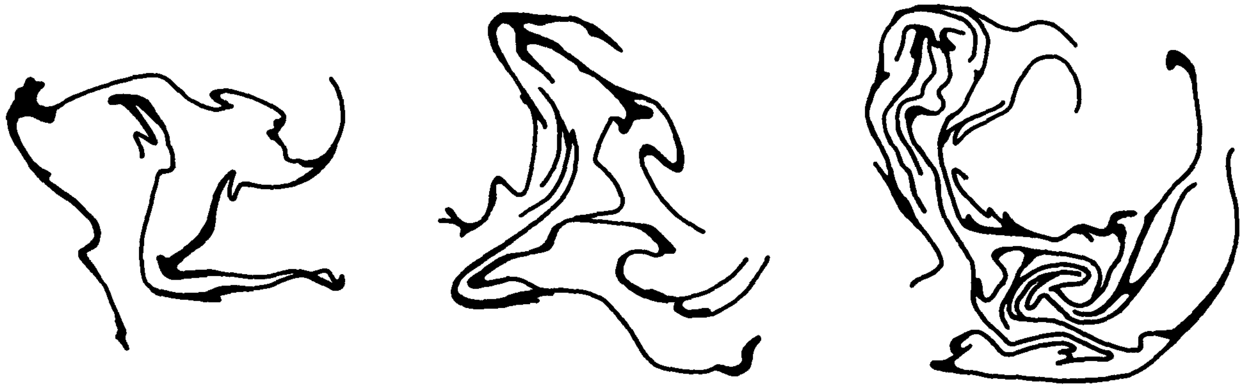
It is useful to rephrase this conclusion. The evolution of the large-scale flow (for which we would like to write deterministic forecast equations) is affected both by the accelerations that are represented by the temporal and advective terms associated with the resolved scales and by the accelerations that for lack of adequate resolution have to be interpreted as the divergence of the unresolvable eddy fluxes of momentum. How do the accelerations of the resolved scales of motion compare with those of the unresolvable scales?

Equations (4) and (7) show quite clearly that two- and three-dimensional flows are fundamentally different in this respect. In the small-scale structure of two-dimensional turbulence (and for scales of motion that are smaller than those usually considered in dynamic meteorology) the accelerations *decrease* as the scale of motion decreases, because $a(l)$ is proportional to l. The accelerations of the resolved scales are dominated by the dynamics of the most energetic large-scale components of the flow field. If the scale of principal forcing is L, those accelerations may be estimated as $\chi^{2/3}L$ by virtue of (4). The accelerations that are caused by the interaction between the resolved and unresolved scales of motion, on the other hand, may be estimated as $\chi^{2/3}l$ if l is the largest unresolved scale of motion. Clearly, the unresolvable scales of motion may be ignored completely if $L \ll l$.

This means that in numerical weather prediction schemes it is permissible to exclude terms representing subgrid scale eddy fluxes, as long as the resolution is sufficient to follow large-scale features and providing that we stay away from those parts of the flow in which three-dimensional turbulence confuses the issue. Also, textbooks on dynamic meteorology do not need to include Reynolds stress terms (eddy fluxes) in the equations used to describe numerical forecasting methods. Even if those terms are not identically zero in those spatially averaged equations, the microstructure cannot generate significant changes in the evolution of the large-scale flow field. The unseen microstructure of the storm sketched in Fig. 2 thus has little influence on its life cycle, and it is fairly harmless to smooth the field of motion in weather map analysis. In this context, we also see why limited area, fine-resolution models may have an edge over models working with a relatively coarse grid: the justification for ignoring unresolved eddy fluxes becomes stronger as the resolution improves.

In three-dimensional turbulence, on the other hand, accelerations *increase* as the scale of motion decreases. Equation (7) states that $a(l)$ is proportional to $l^{-1/3}$; this implies that the very smallest eddies create most of the "noise" in the acceleration field. Fortunately, this problem is not quite so severe in a numerical model with finite resolution, because the accelerations imposed on the resolved scales by the eddy fluxes of the subgrid scale motion are determined primarily by the largest of the unresolved scales (Tennekes and Lumley, 1972, p. 262). Nevertheless, the accelerations suffered by the resolved scales of motion because of the presence of subgrid scale eddy fluxes become *larger* as the resolution is improved!

Since this problem grows worse with improving resolution, turbulence theory (with some notable exceptions) tends to give up altogether: deterministic forecasting of



the evolution of individual eddies is abandoned, and wholesale averaging techniques are employed. The theory of turbulence is not by accident primarily a statistical theory: the opportunities for a deterministic approach are severely limited by the vigor of the microstructure. Turbulence theory is similar in structure to the theory of the general circulation (for an example, see Tennekes (1977)). The latter, however, can draw on a wealth of synoptic experience, while the former has to make do with a quite limited understanding of the dynamics of individual eddies.

The cloud in Fig. 1 can now be viewed with educated eyes. Evidently, we would be naive about its dynamics if we do not sketch it in such a way that the presence of its vigorous microstructure is taken into account. That way, we also are reminded that such a cloud cannot be modeled without a parameterized version of the effects of the unresolved scales of motion.

6. Time scales at small scales of motion

It is common practice in turbulence theory to take the reciprocal of the vorticity whenever an estimate for the characteristic interaction time of any group of eddies is needed. The vorticity represents the angular velocity of a fluid parcel; the reciprocal of the vorticity then is (apart from a factor 2π) an estimate for the "turnover time" of an eddy. This time scale presumably is also proportional to the time required for significant dynamical interactions between an eddy and its immediate environment. The time scale $\tau(l)$ of any scale of motion l such that $l < L$ is found from (2) and (6). In two-dimensional flow, we obtain

$$\tau(l) \sim \chi^{-1/3}, \tag{8}$$

and the corresponding estimate for flow in three dimensions is

$$\tau(l) \sim \epsilon^{-1/3} l^{2/3}. \tag{9}$$

For small eddies in two-dimensional flow and for subsynoptic eddies in the free troposphere the eddy time scale is independent of the scale of motion. In the atmosphere, τ is typically ~ 1 day. This means, for example, that subsynoptic eddies do not evolve much faster than synoptic ones (at least not on the average) and that rapid weather changes at a fixed location tend to be caused by the advection of smaller-scale disturbances by the large-scale flow, not by the rapid evolution of small-scale systems. Still, we have to keep in mind that the occasional very intense subsynoptic eddy may affect the local weather in dramatic ways.

In three-dimensional turbulence the time scales of eddies decrease quite rapidly as the scale of motion decreases. Small eddies have short life spans! The large-scale features of a convective cloud evolve slowly, and many generations of small eddies have evolved before the largest eddies have changed appreciably. The small-scale convolutions on the surface of a cumulus cloud change rapidly, and the task of keeping track of these wrinkles is hopeless, as anyone who has ever watched cumuli drift by can confirm. Again, it makes sense to perform averaging operations. The time scales of interest for most practical purpose are those of the largest scales of motion, and the rapid changes in the microstructure would require excessively short time steps in a numerical model.

According to the theory of predictability in turbulent flows (Leith, 1971; Leith and Kraichnan, 1972), the maximum period over which the evolution of any kind of eddy can be predicted with deterministic methods is on the order of 6–14 time scales of the eddies concerned. The microstructure of three-dimensional turbulence, therefore, is unpredictable *in principle* when time is measured in units that correspond to the time scales characterizing the large-scale features of the flow field. The statistical methods of analysis used in turbulence theory are not just a matter of convenience or computational economy, but one of necessity.

In two-dimensional and atmospheric flows the time scales of small eddies are comparable with those of their larger brothers and sisters. As errors in a numerical forecast propagate from the unresolved scales toward the largest scales of motion, it takes about one time scale to contaminate the solution in each successive octave of the spectrum (Leith and Kraichnan, 1972). Better resolution pays off here, too: if the grid size of a numerical weather prediction scheme is cut in half, it takes another day before the computer output becomes useless. In the atmospheric boundary layer or in convective clouds, on the other hand, only a few seconds or minutes are gained if the resolution is improved by a factor of 2.

7. Conclusion

With a minimum of mathematics, we have explored the principal differences between turbulent flows in two and three dimensions. We have found that the vigorous microstructure of three-dimensional turbulence cannot be handled with deterministic methods and that averaging procedures are necessary, even if they lead to the problem of parameterizing eddy fluxes. In two-dimensional and large-scale atmospheric flows, on the other hand, the microstructure is not capable of influencing the large-scale development significantly, so that there is no need to worry too much about subgrid scale fluxes (except for the friction layer, convective clouds, moisture, and radiation). Nevertheless, the predictability of atmospheric flows is limited; as we venture beyond the maximum predictability period, we enter into the domain of climate dynamics. Climate studies require statistical methods of analysis because the eddy fluxes of the unresolved scales of motion need to be parameterized. Climate models are similar to turbulence models in this respect. Before too long, climate modelers will have much more in common with turbulence modelers and micrometeorologists than either group now seems to realize.

Acknowledgments. The opinions expressed in this paper were developed in part in discussion with several generations of students in my turbulence courses and with my colleagues at the Pennsylvania State University and the University of Washington. R. J. Reid and J. A. Dutton deserve to be mentioned explicitly, because they pointed out that I should not dismiss subsynoptic eddies in an offhand fashion. Research was supported by the Atmospheric Sciences Section of the National Science Foundation with grant DES76-13357.

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Semi-technical report on forecasting/ozone

In 1975 and 1976 scientists at the National Center for Atmospheric Research held two separate forums, one on forecasting and the other on atmospheric ozone. Summaries of those forums have been published in one semi-technical report, which is now available to the public. Experts from a variety of research programs spoke at the forums and their comments have been summarized in the 40-page booklet, "Weather Forecasting/Atmospheric Ozone." The report is designed to give teachers, students, and other interested readers a general overview of two atmospheric questions that are of great importance to society. Single copies are available from: Jan Emery, Publications Office-F2, NCAR, Box 3000, Boulder, Colo. 80307.

NCAR film on ballooning available

A new film on scientific ballooning is available from the National Center for Atmospheric Research. NCAR's National Scientific Balloon Facility (NSBF) in Palestine, Tex., is the site for balloon launches by researchers from around the world who are performing experiments in the stratospheric altitudes. NSBF, one of the few balloon launching facilities in the world, has the capability of launching payloads of scientific instruments weighing a ton or more to altitudes of 36 576 m or higher.

The film follows the preparation of scientific experiments for their flight from the balloon facility, through the launch and recovery of the balloon payload, providing an overview of scientific ballooning today. It is in color and is 25 minutes long. High school and college science classes and audiences interested in an introduction to some aspect of atmospheric research will benefit from the film, which can be rented from NCAR for a nominal fee. Inquiries should be addressed to: NCAR Films, P.O. Box 3000, Boulder, Colo. 80307.

Other films available from NCAR include: "Chemistry above the Clouds"; "Explosions on the Sun"; "51 dBZ—The National Hail Research Experiment"; "Ice in the Atmosphere"; "Shadow across the Sun"; and "Two-Niner Juliet."

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